Augusta Christian Schools

AP Calculus

Summer Work

Mrs. Stephanie Kellogg

Please follow all the instructions. Failure to follow all directions will result in a zero on this assignment.

1. This is a college level course, and the expectation is that you rise to the challenge and demonstrate a college level work ethic and quality work .
2. You must show all of your work for each problem.
3. You must write all the work neatly in the space provided for each problem.
4. You may not use google or any other math-based software to solve the problems.
5. You may use google only to review concepts.
6. Do not wait until the last minute to complete this packet.
7. This is an individual assignment. Please complete on your own and to the best of your ability.
8. At the end of the assignment, please rate the difficulty level and sign the printed statement.
9. Be sure your name is on the assignment.
10. If you have any questions, email me (stephaniekellogg@augustachristian.org)!
11. Turn the assignments in to Mrs. Kellogg **on the first day of school, August 5.**

Student's Name: _____

Algebra Review

A. Factoring. Factor by removing a GCF.

1. $3x^4 + 4x^3 - x^2$

2. $e^{-x} - xe^{-x} + 2x^2e^{-x}$

3. $2\sqrt{x} + 6x^{\frac{3}{2}}$

4. $\sin x + \tan x$

5. $3\left(x^2 + 4\right)\left(x^2 + 1\right) + 6\left(x^2 + 4\right)^2$

B. Factor the expression.

6. $x^4 - 7x^2 + 12$

7. $e^{2x} + 2 + e^{-2x}$

8. $1 - x^2$

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C. Simplifying. Rewrite the expression in simplest form.

9. $\frac{3x+9}{6x}$

10. $\frac{x^2}{x^{\frac{1}{2}}}$

11. $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

12. $\frac{\sqrt{x^2+1}-\frac{1}{\sqrt{x^2+1}}}{x^2+1}$

13. $\frac{x^2-5x+6}{x^2-4x+4}$

14. $\frac{1}{x+1}-\frac{1}{x-1}-\frac{2}{x^2-1}$

D. Rationalizing. Rationalize by multiplying by the conjugate of the denominator.

15. $\frac{1}{1-\cos\cos x}$

16. $\frac{x}{1-\sqrt{x^2+1}}$

17. $\frac{2}{x+\sqrt{x^2+1}}$

Complete pg. 56 #1-43 odd

Chapter 1 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–14, write an equation for the specified line.

1. through $(1, -6)$ with slope 3
2. through $(-1, 2)$ with slope $-1/2$
3. the vertical line through $(0, -3)$
4. through $(-3, 6)$ and $(1, -2)$
5. the horizontal line through $(0, 2)$
6. through $(3, 3)$ and $(-2, 5)$
7. with slope -3 and y-intercept 3
8. through $(3, 1)$ and parallel to $2x - y = -2$
9. through $(4, -12)$ and parallel to $4x + 3y = 12$
10. through $(-2, -3)$ and perpendicular to $3x - 5y = 1$
11. through $(-1, 2)$ and perpendicular to $\frac{1}{2}x + \frac{1}{3}y = 1$
12. with x-intercept 3 and y-intercept -5
13. the line $y = f(x)$, where f has the following values:

x	-2	2	4
$f(x)$	4	2	1

14. through $(4, -2)$ with x-intercept -3

In Exercises 15–18, determine whether the graph of the function is symmetric about the y-axis, the origin, or neither.

15. $y = x^{1/5}$
16. $y = x^{2/5}$
17. $y = x^2 - 2x - 1$
18. $y = e^{-x^2}$

In Exercises 19–26, determine whether the function is even, odd, or neither.

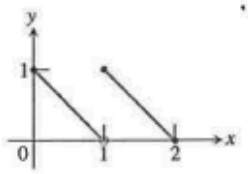
19. $y = x^2 + 1$
20. $y = x^5 - x^3 - x$
21. $y = 1 - \cos x$
22. $y = \sec x \tan x$
23. $y = \dfrac{x^4 + 1}{x^3 - 2x}$
24. $y = 1 - \sin x$
25. $y = x + \cos x$
26. $y = \sqrt{x^4 - 1}$

In Exercises 27–38, find the (a) domain and (b) range, and (c) graph the function.

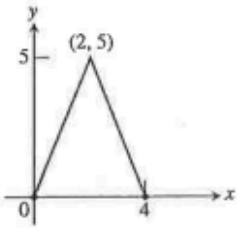
27. $y = |x| - 2$
28. $y = -2 + \sqrt{1 - x}$
29. $y = \sqrt{16 - x^2}$
30. $y = 3^{2-x} + 1$
31. $y = 2e^{-x} - 3$
32. $y = \tan(2x - \pi)$
33. $y = 2\sin(3x + \pi) - 1$
34. $y = x^{2/5}$
35. $y = \ln(x - 3) + 1$
36. $y = -1 + \sqrt[3]{2 - x}$
37. $y = \begin{cases} \sqrt{-x}, & -4 \le x \le 0 \\ \sqrt{x}, & 0 < x \le 4 \end{cases}$
38. $y = \begin{cases} -x - 2, & -2 \le x \le -1 \\ x, & -1 < x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$

In Exercises 39 and 40, write a piecewise formula for the function.

39.



40.



In Exercises 41 and 42, find

(a) $(f \circ g)(-1)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(x)$ (d) $(g \circ g)(x)$

41. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{\sqrt{x+2}}$
42. $f(x) = 2 - x, \quad g(x) = \sqrt[3]{x+1}$

In Exercises 43 and 44, (a) write a formula for $f \circ g$ and $g \circ f$ and find the (b) domain and (c) range of each.

43. $f(x) = 2 - x^2, \quad g(x) = \sqrt{x+2}$
44. $f(x) = \sqrt{x}, \quad g(x) = \sqrt{1-x}$

In Exercises 45–48, a parametrization is given for a curve.

(a) Graph the curve. Identify the initial and terminal points, if any. Indicate the direction in which the curve is traced.

(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

45. $x = 5\cos t, \quad y = 2\sin t, \quad 0 \le t \le 2\pi$
46. $x = 4\cos t, \quad y = 4\sin t, \quad \pi/2 \le t < 3\pi/2$
47. $x = 2 - t, \quad y = 11 - 2t, \quad -2 \le t \le 4$
48. $x = 1 + t, \quad y = \sqrt{4 - 2t}, \quad t \le 2$

In Exercises 49–52, give a parametrization for the curve.

49. the line segment with endpoints $(-2, 5)$ and $(4, 3)$
50. the line through $(-3, -2)$ and $(4, -1)$
51. the ray with initial point $(2, 5)$ that passes through $(-1, 0)$
52. $y = x(x - 4), \quad x \le 2$

Group Activity In Exercises 53 and 54, do the following.

(a) Find f^{-1} and show that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.
(b) Graph f and f^{-1} in the same viewing window.

53. $f(x) = 2 - 3x$
54. $f(x) = (x + 2)^2, \quad x \ge -2$

In Exercises 55 and 56, find the measure of the angle in radians and degrees.

55. $\sin^{-1}(0.6)$
56. $\tan^{-1}(-2.3)$
57. Find the six trigonometric values of $\theta = \cos^{-1}(3/7)$. Give exact answers.
58. Solve the equation $\sin x = -0.2$ in the following intervals.

(a) $0 \le x < 2\pi$ (b) $-\infty < x < \infty$

59. Solve for x: $e^{-0.2x} = 4$

Complete Pg. D26-D27 #19, 21, 27-39 odd, 51-54

In Exercises 19–22, evaluate the sine, cosine, and tangent of each angle *without* using a calculator.

19. (a) $60°$
(b) $120°$
(c) $\frac{\pi}{4}$
(d) $\frac{5\pi}{4}$

20. (a) $-30°$
(b) $150°$
(c) $-\frac{\pi}{6}$
(d) $\frac{\pi}{2}$

21. (a) $225°$
(b) $-225°$
(c) $\frac{5\pi}{3}$
(d) $\frac{11\pi}{6}$

22. (a) $750°$
(b) $510°$
(c) $\frac{10\pi}{3}$
(d) $\frac{17\pi}{3}$

In Exercises 23–26, use a calculator to evaluate the trigonometric functions to four significant digits.

23. (a) $\sin 10°$
(b) $\csc 10°$

24. (a) $\sec 225°$
(b) $\sec 135°$

25. (a) $\tan\frac{\pi}{9}$
(b) $\tan\frac{10\pi}{9}$

26. (a) $\cot(1.35)$
(b) $\tan(1.35)$

In Exercises 27–30, find two solutions of each equation. Express the results in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

27. (a) $\cos\theta = \frac{\sqrt{2}}{2}$
(b) $\cos\theta = -\frac{\sqrt{2}}{2}$

28. (a) $\sec\theta = 2$
(b) $\sec\theta = -2$

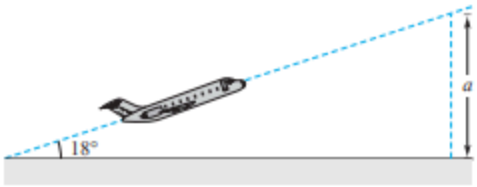
29. (a) $\tan\theta = 1$
(b) $\cot\theta = -\sqrt{3}$

30. (a) $\sin\theta = \frac{\sqrt{3}}{2}$
(b) $\sin\theta = -\frac{\sqrt{3}}{2}$

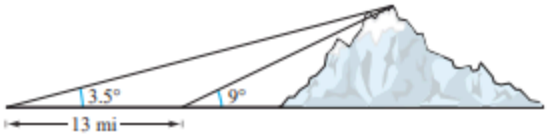
In Exercises 31–38, solve the equation for θ ($0 \le \theta < 2\pi$).

31. $2\sin^2\theta = 1$
32. $\tan^2\theta = 3$
33. $\tan^2\theta - \tan\theta = 0$
34. $2\cos^2\theta - \cos\theta = 1$
35. $\sec\theta\csc\theta = 2\csc\theta$
36. $\sin\theta = \cos\theta$
37. $\cos^2\theta + \sin\theta = 1$
38. $\cos\frac{\theta}{2} - \cos\theta = 1$

39. ***Airplane Ascent*** An airplane leaves the runway climbing at $18°$ with a speed of 275 feet per second (see figure). Find the altitude a of the plane after 1 minute.



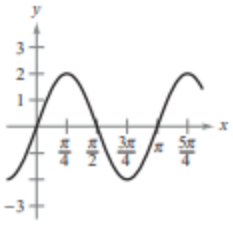
40. ***Height of a Mountain*** In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is $3.5°$. After you drive 13 miles closer to the mountain, the angle of elevation is $9°$. Approximate the height of the mountain.

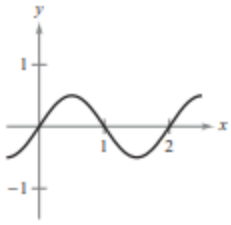


(not to scale)

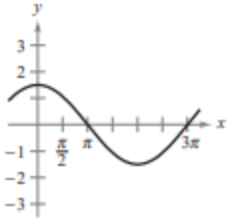
In Exercises 41–44, determine the period and amplitude of each function.

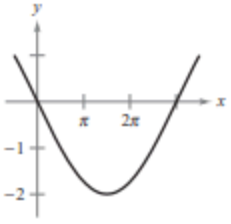
41. (a) $y = 2\sin 2x$
(b) $y = \frac{1}{2}\sin\pi x$





42. (a) $y = \frac{3}{2}\cos\frac{x}{2}$
(b) $y = -2\sin\frac{x}{3}$





43. $y = 3\sin 4\pi x$

44. $y = \frac{2}{3}\cos\frac{\pi x}{10}$

In Exercises 45–48, find the period of the function.

45. $y = 5 \tan 2x$

46. $y = 7 \tan 2\pi x$

47. $y = \sec 5x$

48. $y = \csc 4x$

Writing **In Exercises 49 and 50, use a graphing utility to graph each function f on the same set of coordinate axes for $c = -2$, $c = -1$, $c = 1$, and $c = 2$. Give a written description of the change in the graph caused by changing c.**

49. (a) $f(x) = c \sin x$
(b) $f(x) = \cos(cx)$
(c) $f(x) = \cos(\pi x - c)$

50. (a) $f(x) = \sin x + c$
(b) $f(x) = -\sin(2\pi x - c)$
(c) $f(x) = c \cos x$

In Exercises 51–62, sketch the graph of the function.

51. $y = \sin\dfrac{x}{2}$

52. $y = 2 \cos 2x$

53. $y = -\sin\dfrac{2\pi x}{3}$

54. $y = 2 \tan x$

55. $y = \csc\dfrac{x}{2}$

56. $y = \tan 2x$

57. $y = 2 \sec 2x$

58. $y = \csc 2\pi x$

59. $y = \sin(x + \pi)$

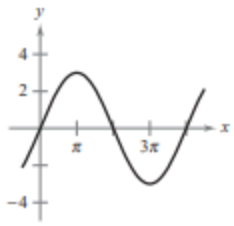
60. $y = \cos\left(x - \dfrac{\pi}{3}\right)$

61. $y = 1 + \cos\left(x - \dfrac{\pi}{2}\right)$

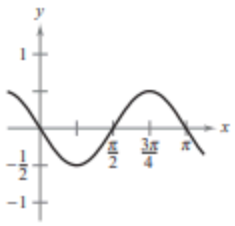
62. $y = 1 + \sin\left(x + \dfrac{\pi}{2}\right)$

Graphical Reasoning **In Exercises 63 and 64, find a, b, and c such that the graph of the function matches the graph in the figure.**

63. $y = a \cos(bx - c)$



64. $y = a \sin(bx - c)$



65. ***Think About It*** Sketch the graphs of $f(x) = \sin x$, $g(x) = |\sin x|$, and $h(x) = \sin(|x|)$. In general, how are the graphs of $|f(x)|$ and $f(|x|)$ related to the graph of f?

66. ***Think About It*** The model for the height h of a Ferris wheel car is

$$h = 51 + 50 \sin 8\pi t$$

where t is measured in minutes. (The Ferris wheel has a radius of 50 feet.) This model yields a height of 51 feet when $t = 0$. Alter the model so that the height of the car is 1 foot when $t = 0$.

67. ***Sales*** Sales S, in thousands of units, of a seasonal product are modeled by

$$S = 58.3 + 32.5 \cos\frac{\pi t}{6}$$

where t is the time in months (with $t = 1$ corresponding to January and $t = 12$ corresponding to December). Use a graphing utility to graph the model for S and determine the months when sales exceed 75,000 units.

68. ***Investigation*** Two trigonometric functions f and g have a period of 2, and their graphs intersect at $x = 5.35$.

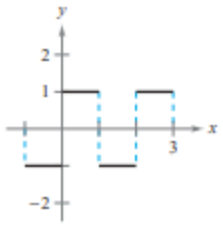
(a) Give one smaller and one larger positive value of x where the functions have the same value.

(b) Determine one negative value of x where the graphs intersect.

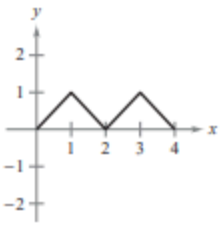
(c) Is it true that $f(13.35) = g(-4.65)$? Give a reason for your answer.

Pattern Recognition **In Exercises 69 and 70, use a graphing utility to compare the graph of f with the given graph. Try to improve the approximation by adding a term to $f(x)$. Use a graphing utility to verify that your new approximation is better than the original. Can you find other terms to add to make the approximation even better? What is the pattern? (In Exercise 69, sine terms can be used to improve the approximation and in Exercise 70, cosine terms can be used.)**

69. $f(x) = \dfrac{4}{\pi}\left(\sin \pi x + \dfrac{1}{3}\sin 3\pi x\right)$



70. $f(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2}\left(\cos \pi x + \dfrac{1}{9}\cos 3\pi x\right)$



On a scale from 1 to 10 (1 being extremely easy and 10 being extremely difficult), how difficult was this summer assignment? _____

Statement: "I did this work on my own and did not copy answers from any source."

Signed: _____